



## WHAT COMES NEXT: IDEAS AND BRAINSTORMING FOR THE PHD THESIS

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# Outline

## Clinical problems

1. Diffeomorphic image registration for pre-term preadolescent rabbit brain imaging, cross sectional studies.
2. Mosaicing of fetoscopic videos: distortion correction.
3. Landmark tracking in fetoscopic placenta examination.

## Methodological problems

4. Efficient computation of the log-composition and Lie logarithm map with improved parallel transport formula.
5. Computation of the Lie exponential: exponential integrators applied to stochastic deformation models.
6. Computation of the Lie exponential: EDT RK4, in medical imaging.
7. Discrete optimisation methods (MRF) to optimise SVF-based energy function.

## Rough idea:

Aim: Analysis of fetal brain MR images with a registration based approach.

Initial idea: Multiple image registration techniques applied for motion correction, resolution improvement, longitudinal and cross sectional studies.

Possible outcome: Differentiating between normal and abnormal brain development.

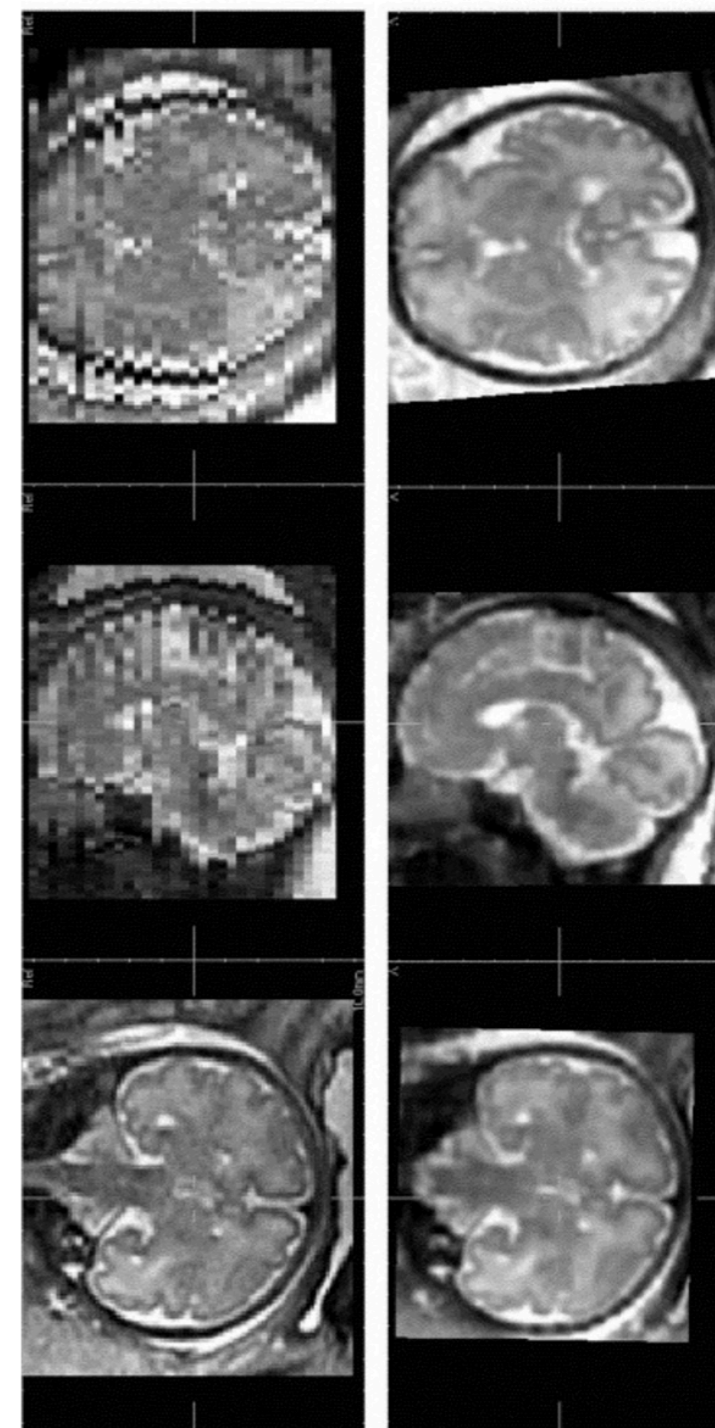
...Details and a better definition of the project and feasibility depends on the data collections...

## Possible initial resources:

[1] Rousseau, Francois, et al. "Registration-based approach for reconstruction of high-resolution in utero fetal MR brain images." *Academic radiology* 13.9 (2006): 1072-1081.

[2] Studholme, Colin. "Mapping fetal brain development in utero using MRI: the big bang of brain mapping." *Annual review of biomedical engineering* 13 (2011): 345.

[3] Gholipour, Ali, et al. "Fetal brain volumetry through MRI volumetric reconstruction and segmentation." *International journal of computer assisted radiology and surgery* 6.3 (2011): 329-339.



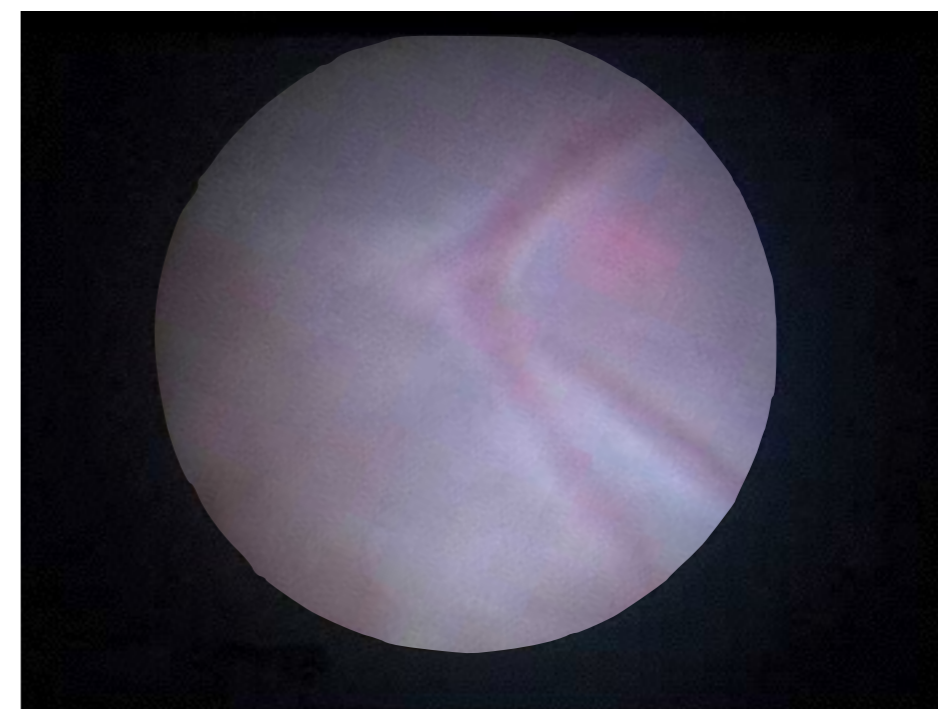
## Mosaicing steps (Marcel and Djoshkun WBIR paper):

1. Features detector (Laplacian based)
2. Features descriptor (SIFT)
3. Point sets association with random sampling consensus (RANSAC)
4. Probabilistic models: random noise is added to each homographic transformation.
5. Mosaicing composition based on the resulting homographies.
6. Multi band blending algorithm.



## Distortion correction:

Frame to reference parameters information can be considered with a frame to frame registration. Parameters of the first can be corrected with the information obtained from the latter.



Possible way to correct distortion:

homographies transformation between frames, parametrised in the tangent space (here the error is introduced):

$$h_1^2 = \log(H_1^2)$$

$$\tilde{h}_1^2 = h_1^2 + \epsilon_1^2$$

$$\tilde{H}_1^2 = \exp(h_1^2 + \epsilon_1^2)$$

$$\hat{\epsilon} = (\epsilon_k^{k+1})_{k=1}^{N-1} \in \text{Hom}^{N-1}$$

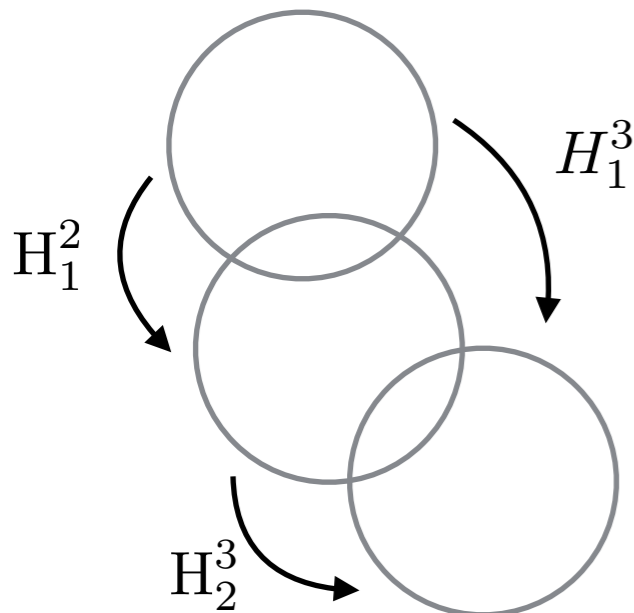
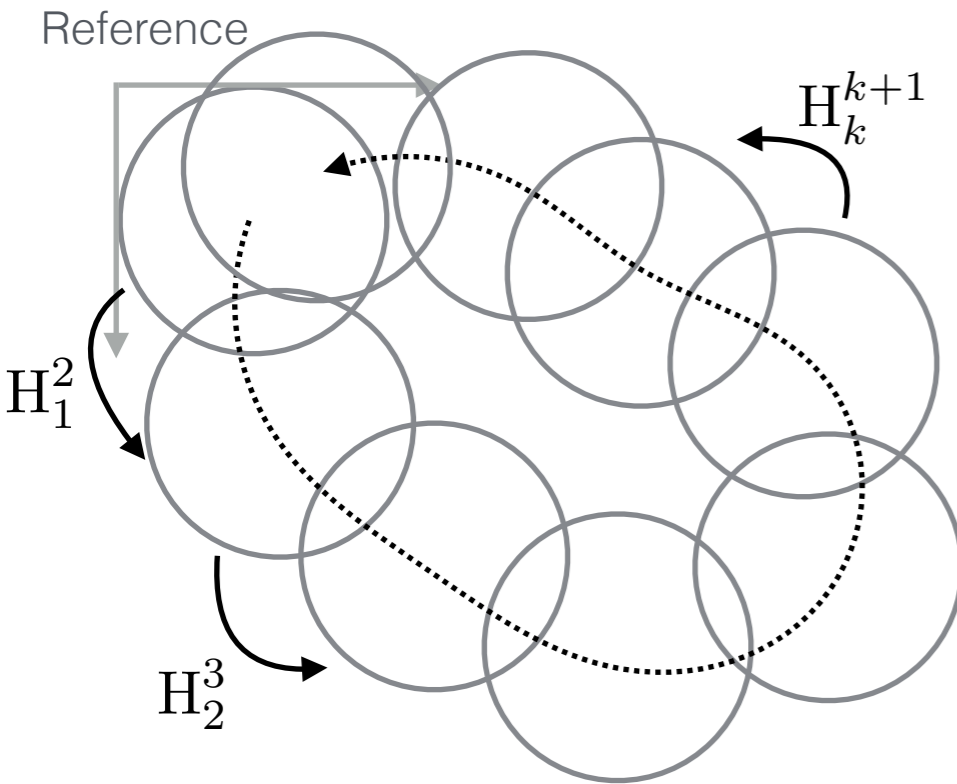
After all the homographies are computed with their error (first coarse phase), a refinement may be obtained optimising:

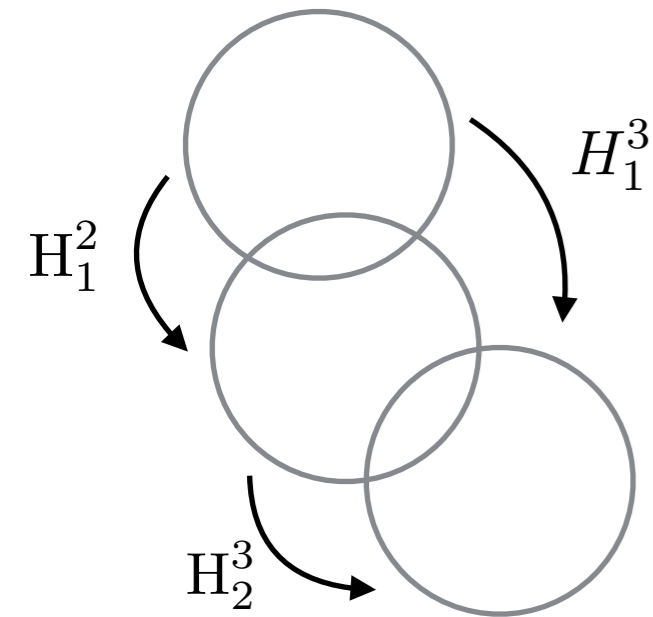
$$\hat{\epsilon} = \underset{\epsilon}{\operatorname{argmin}} \sum_{k=1}^{N-1} d(f_k \circ F_k, F_{k+1})$$

where  $d$  is a distance between the two frames (that take into account only the overlap regions e.g. NMI), and  $f_k$  is the frame to reference map, that accumulates all the errors.

$$\begin{aligned} f_k &= \tilde{H}_{k-1}^k \circ \dots \circ \tilde{H}_2^3 \circ \tilde{H}_1^2 \\ &= \exp(h_{k-1}^k + \epsilon_{k-1}^k) \circ \dots \circ \exp(h_2^3 + \epsilon_2^3) \circ \exp(h_1^2 + \epsilon_1^2) \end{aligned}$$

The values that each of the error can assume can be chose ad hoc (small increase in the range for each new frame, or guided by the features descriptors registration). This may increase the accuracy of the final results.





Alternatively, the error can be modelled in the composition of homographies:

$$\begin{aligned}\tilde{h}_1^3 &= \log(\exp(h_2^3) \circ \exp(h_1^2)) + \epsilon_1^3 \\ \tilde{h}_2^4 &= \log(\exp(h_3^4) \circ \exp(h_2^3)) + \epsilon_3^4\end{aligned}$$

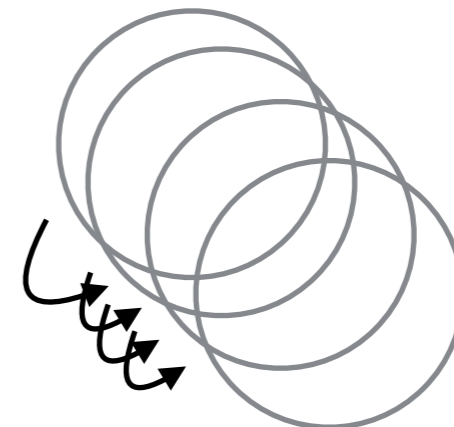
Previous optimisation problem becomes:

$$\hat{\epsilon} = \underset{\epsilon}{\operatorname{argmin}} \sum_{k=1}^{N-2} d\left(f_{k+2} \circ F_k, F_{k+2}\right)$$

A third way to model the error, is considering that it is accumulated each time we move away from the first frame. For example, the parameters of the homography, with the cumulation error, from the first to the fourth frame are:

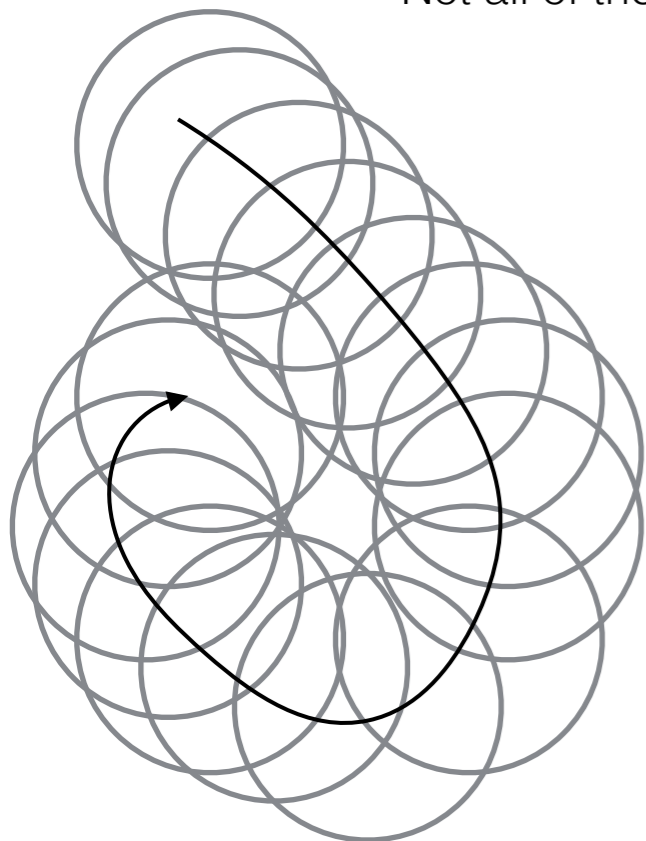
$$\tilde{h}_1^4 = h_3^4 \oplus (h_2^3 \oplus (h_1^2 + \epsilon_1^2) + \epsilon_1^3) + \epsilon_1^4$$

These approaches do not take into account information given by multiple overlaps.  
Overlaps are considered only for consecutive pairs of frames...



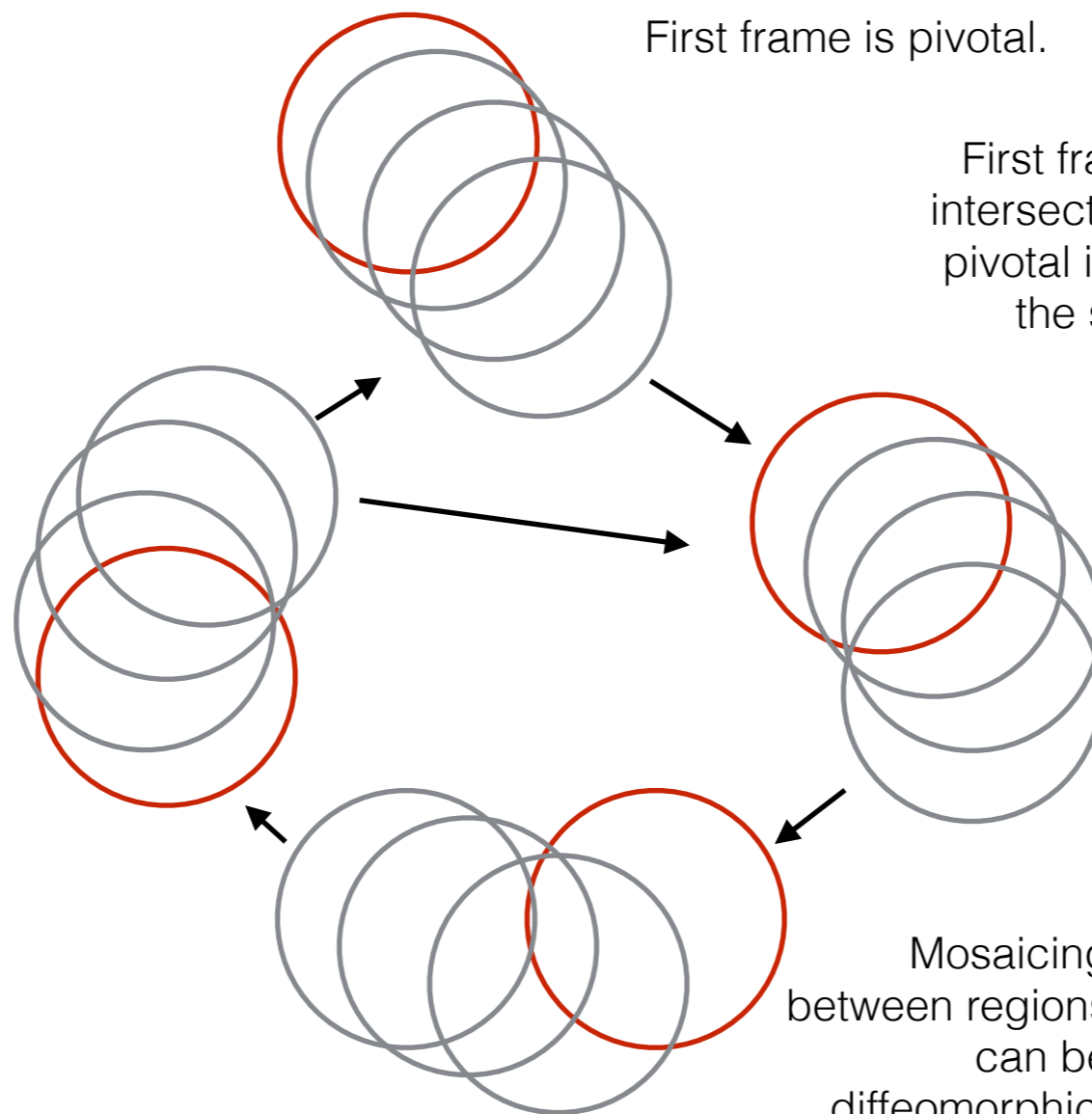
Not all of the frames have the same importance...

We can identify *pivotal frames*, respect to other frames, and *pivotal regions* respect to other regions.



First frame is pivotal.

First frame with no intersection with the pivotal is pivotal for the subsequent frames.



Each pivot defines a pivotal region.

inter-regional mosaicing can be performed with projective registration.

Mosaicing between regions can be diffeomorphic.

... Or a probabilistic model can be applied, with only one parameter for each couple of intersecting region, instead of one parameter per frames.

Does it make sense?

The new problem is the determination of overlapping regions between any couple of frames.

## Possible initial resources:

[1] Bourmaud, Guillaume, et al. "From Intrinsic Optimization to Iterated Extended Kalman Filtering on Lie Groups." *Journal of Mathematical Imaging and Vision* (2015): 1-20.

[2] Vercauteren, Tom, et al. "Robust mosaicing with correction of motion distortions and tissue deformations for in vivo fibered microscopy." *Medical image analysis* 10.5 (2006): 673-692.

[3] Sawhney, Harpreet S., Steve Hsu, and Rakesh Kumar. "Robust video mosaicing through topology inference and local to global alignment." *Computer Vision—ECCV'98*. Springer Berlin Heidelberg, 1998. 103-119.

[4] Shum, Heung-Yeung, and Richard Szeliski. "Systems and experiment paper: Construction of panoramic image mosaics with global and local alignment." *International Journal of Computer Vision* 36.2 (2000): 101-130.

[5] Szeliski, Richard. "Image alignment and stitching: A tutorial." *Foundations and Trends® in Computer Graphics and Vision* 2.1 (2006): 1-104.

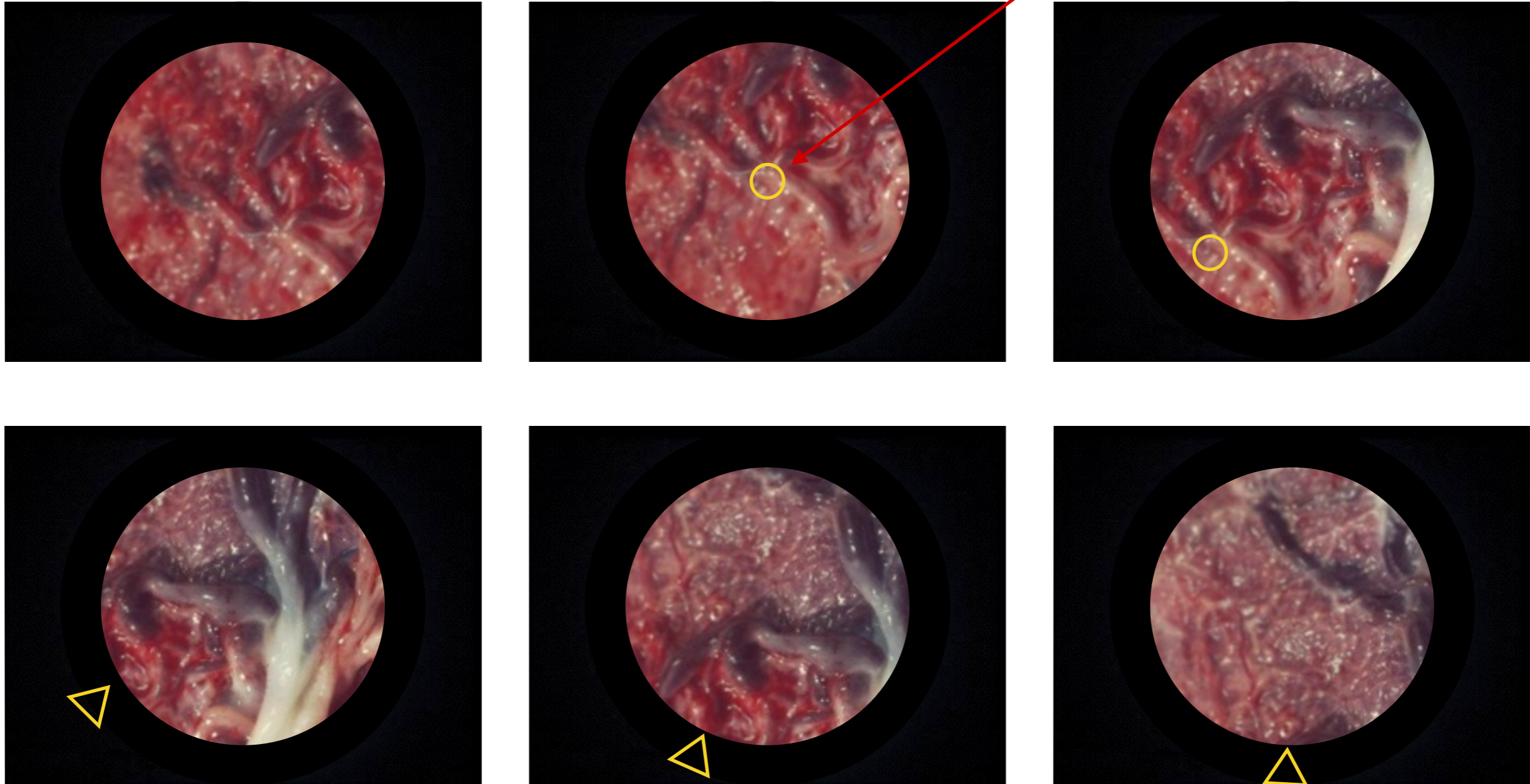


Rough idea:

Problem: no absolute reference for the surgeon while exploring the placenta.

What if the surgeon can select a point of interest that remains visible in the fetoscopic camera (and in the final mosaicing)?

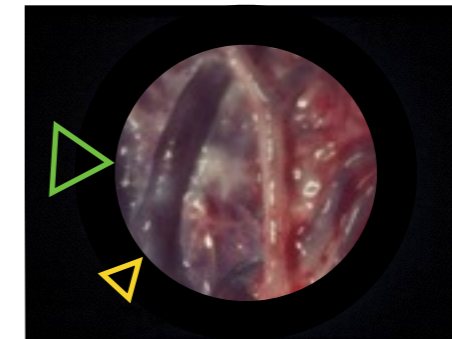
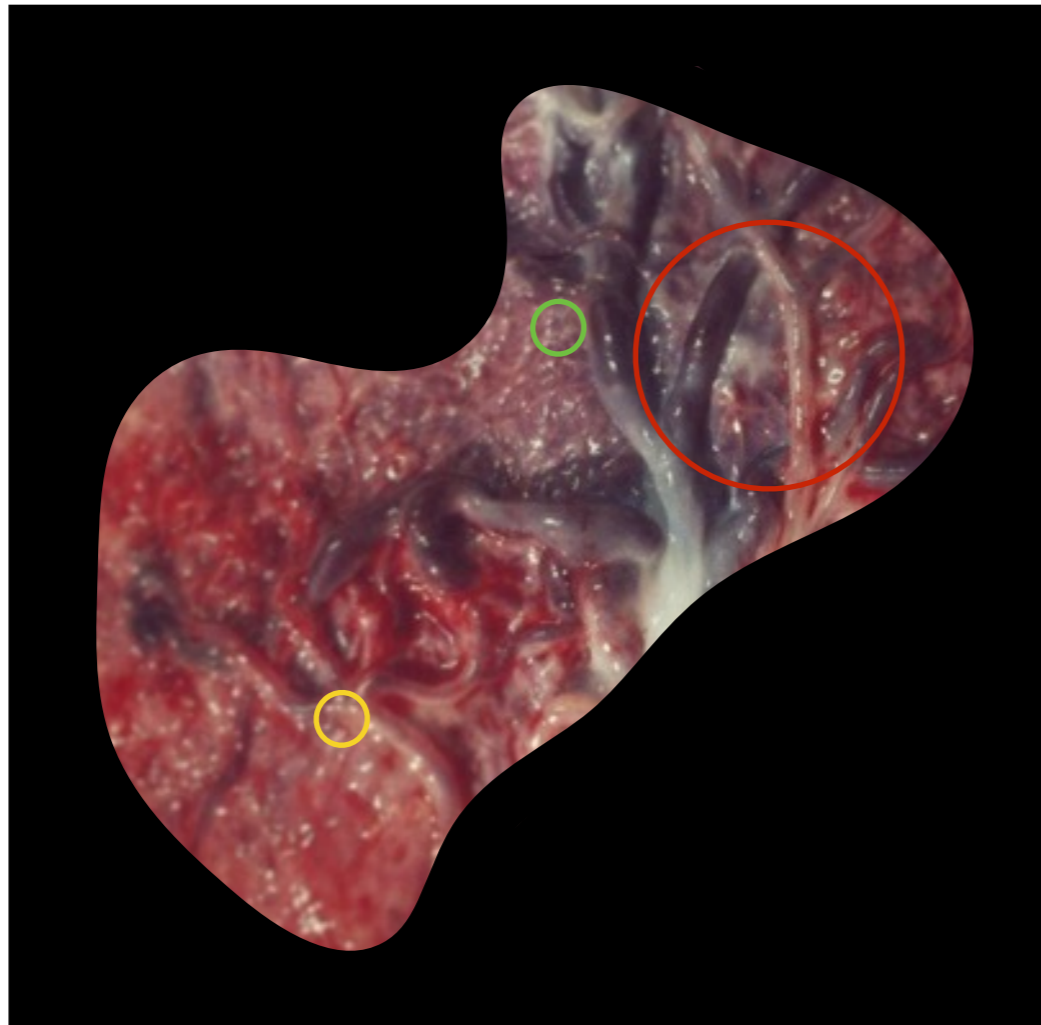
The point is selected on the screen by the surgeon, using a pushbutton on the fetoscope.



The visual reference remains on the fetoscope view. The surgeon knows where he is respect to the selected point.

Ideally... There can be more than one tracking point.

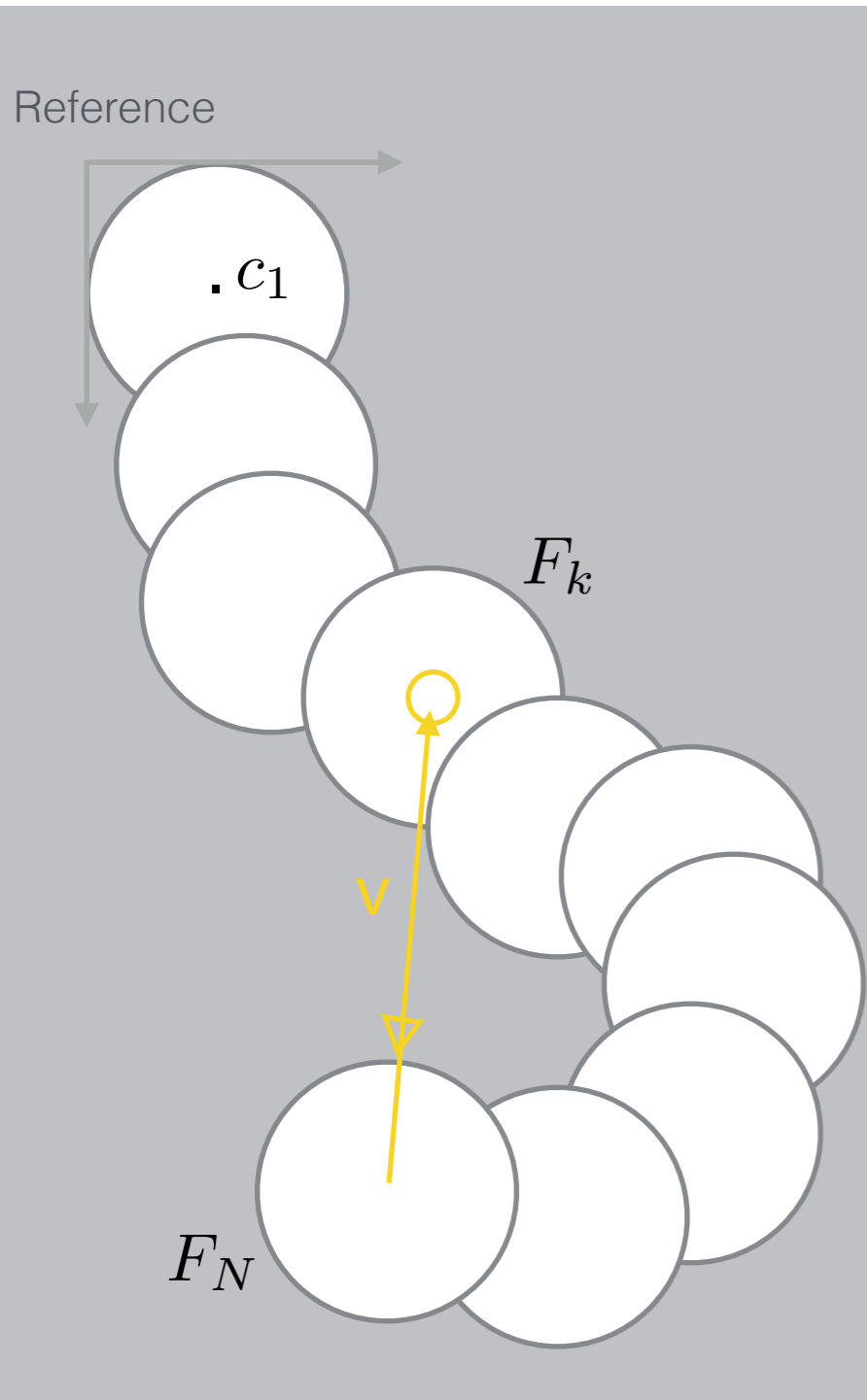
...all the selected points will appear in the final mosaic, or are in a partial mosaic visualised in real time besides the screen with the fetoscope image.



The size of the reference provides an idea of the distance.

...all of the points that are laser-coagulated can be automatically introduced as point of interests in the final mosaic.

(Other useful information during the procedure is the distance of the fetoscope from the placenta and the consequent power intensity of the laser. Can this be recovered from the image blur?)

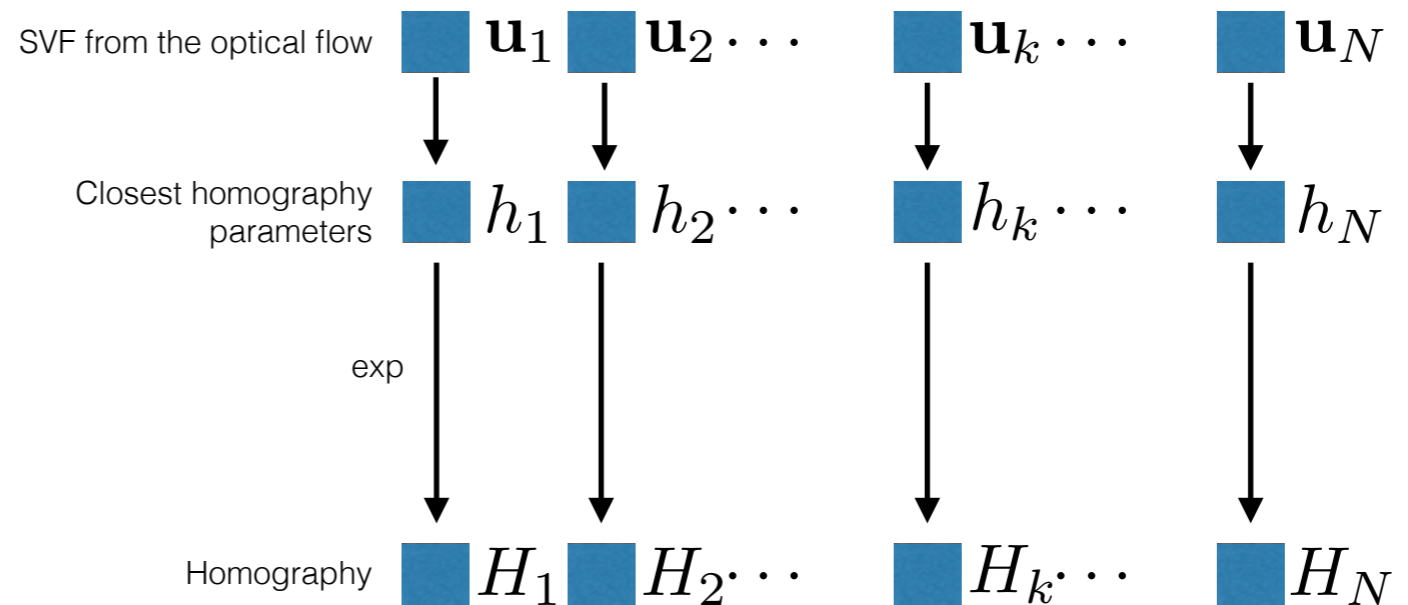


1. At each frame

- Compute the optical flow (SVF) from the motion (wavelet motion model to estimate the optical flow?)
- Obtain the homography of the motion of the center of the camera as the closest homography to the SVF obtained before (Hilbert projection theorem?)

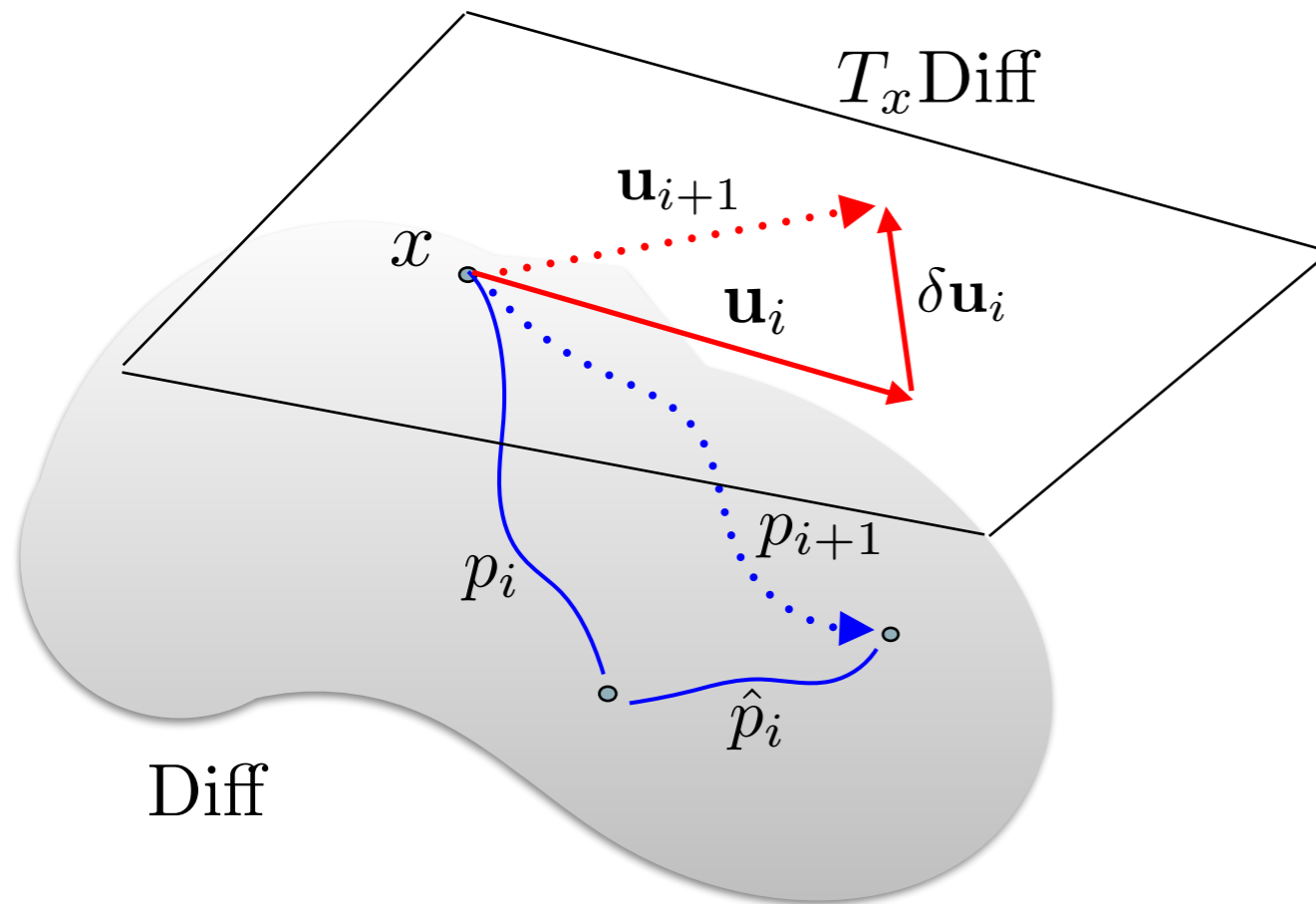
An option to compute the vector from the tracked point, center of the frame K to the center of the current frame is

$$v = H_1^k c_1 - H_1^N c_1$$



## Possible initial resources:

- [1] Uyttendaele, Matthew, Ashley Eden, and Richard Skeliski. "Eliminating ghosting and exposure artifacts in image mosaics." *Computer Vision and Pattern Recognition, 2001. CVPR 2001. Proceedings of the 2001 IEEE Computer Society Conference on*. Vol. 2. IEEE, 2001.
- [2] Peleg, Shmuel, et al. "Mosaicing on adaptive manifolds." *Pattern Analysis and Machine Intelligence, IEEE Transactions on* 22.10 (2000): 1144-1154.
- [3] Davis, James. "Mosaics of scenes with moving objects." *Computer Vision and Pattern Recognition, 1998. Proceedings. 1998 IEEE Computer Society Conference on*. IEEE, 1998.
- [4] Vercauteren, Tom, et al. "Mosaicing of confocal microscopic in vivo soft tissue video sequences." *Medical Image Computing and Computer-Assisted Intervention–MICCAI 2005*. Springer Berlin Heidelberg, 2005. 753-760.
- [5] Barron, John L., David J. Fleet, and Steven S. Beauchemin. "Performance of optical flow techniques." *International journal of computer vision* 12.1 (1994): 43-77.
- [6] Caballero, Fernando, et al. "Homography based Kalman filter for mosaic building. applications to UAV position estimation." *Robotics and Automation, 2007 IEEE International Conference on*. IEEE, 2007.



Missing step: proof of the assumption at the base of the parallel transport concept.

Main idea:

Continue the work started in the MRes for the log composition, exploiting the quick computation of the exponential.

$$\begin{aligned} \mathbf{u}_{i+1} &= BCH(\mathbf{u}_i, \delta \mathbf{u}_i) \\ &= \log(\exp(\mathbf{u}_i) \circ \exp(\delta \mathbf{u}_i)) \end{aligned}$$

Use the improved log composition (if any), to compute the logarithm map via the algorithm proposed by Bossa.

$$\begin{cases} \mathbf{u}_0 = 0 \\ \mathbf{u}_{j+1} = BCH^k(\mathbf{u}_j, \text{app}(-\mathbf{u}_j \oplus \mathbf{u})) \end{cases}$$

(Bibliography: See MRes)

Rough idea: Deformation model defined by solutions of stochastic differential equations

$$\dot{\mathbf{x}}(t) = \mathbf{v}(\mathbf{x}(t)) \longrightarrow \dot{\mathbf{x}}(t) = \mathbf{v}(\mathbf{x}(t)) + \epsilon_t \mathbf{w}(\mathbf{x}(t))$$

...And consequent application to exponential integrators for the computation of the Lie exponential map.

## Initial bibliography:

Elworthy, K. D. "Stochastic dynamical systems and their flows." Stochastic analysis (1978): 79-95.

Sidorov, Kirill A., Stephen Richmond, and David Marshall. "An efficient stochastic approach to groupwise non-rigid image registration." Computer Vision and Pattern Recognition, 2009. CVPR 2009. IEEE Conference on. IEEE, 2009.

Sai-Kit, Yeung, et al. "Enforcing stochastic inverse consistency in non-rigid image registration and matching." Computer Vision and Pattern Recognition, 2008. CVPR 2008. IEEE Conference on. IEEE, 2008.

ALBEVERIO, SERGIO, ALEXEI DALETSKII, and YURI KONDRATIEV. "A stochastic differential equation approach to some lattice models on compact Lie groups." Random Operators and Stochastic Equations 4.3 (1996): 239-250.

Budhiraja, Amarjit, Paul Dupuis, and Vasileios Maroulas. "Large deviations for stochastic flows of diffeomorphisms." Bernoulli 16.1 (2010): 234-257.

Wassermann, Demian, et al. "Probabilistic diffeomorphic registration: Representing uncertainty." Biomedical Image Registration. Springer International Publishing, 2014. 72-82.

Bichteler, Klaus. Stochastic integration with jumps. Vol. 89. Cambridge University Press, 2002.

## Main idea:

Reimplement the Exponential Time Differencing methods for the computation of the exponential map.  
Continue the WBIR submission, following the algorithms proposed by Cox Matthews (2002).

$$\dot{\mathbf{x}}(t) = \mathbf{v}_0 + \mathbf{J}_{\mathbf{v}_0} \mathbf{x}(t) + \mathcal{N}_{\mathbf{v}}(\mathbf{x}(t))$$

Exact integration of the Linear part

+

Numerical integration of the non-Linear part

Interesting results may arise from the combination of the Exponential integrators with the stochastic approach. (Stochastic term in the non-linear part, Ito numerical integration).

## Initial bibliography:

Hochbruck, Marlis, and Alexander Ostermann. "Exponential integrators." Acta Numer 19 (2010): 209-286.

Cox, S. M., and P. C. Matthews. "Exponential time differencing for stiff systems." Journal of Computational Physics 176.2 (2002): 430-455.

## Main idea:

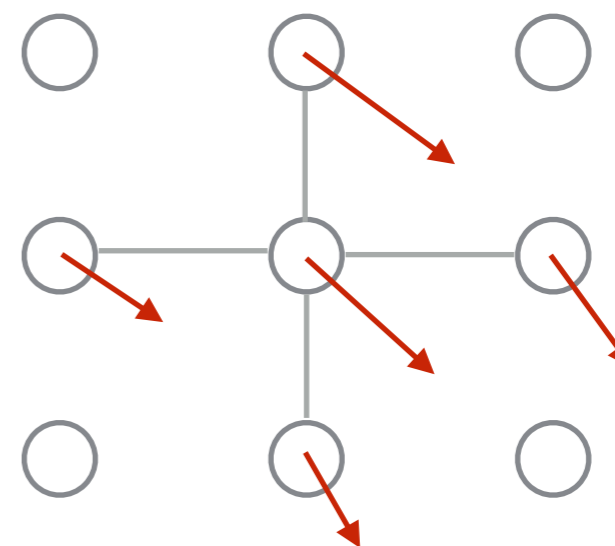
Avoid local minima in the optimisation phase of the image registration algorithm.

First initial point for the continuous optimisation can be provided by the solution of the associated discretised problem, to reduce the occurrence of local minima.

## Model:

Control points and their mutual relationships are modelled with an undirected graph.

*Discrete* labels of vertices represents the values of the displacement at each point, discrete labels of the edges penalises the deviation of neighbouring control point.



## Initial bibliography:

Rangarajan, Anand, and Rama Chellappa. "Markov random field models in image processing." (1995).

Heinrich, H. P., et al. "MRF-based deformable registration and ventilation estimation of lung CT." *Medical Imaging, IEEE Transactions on* 32.7 (2013): 1239-1248.

Glocker, Ben, et al. "Dense image registration through MRFs and efficient linear programming." *Medical image analysis* 12.6 (2008): 731-741.

Glocker, Ben, et al. "Deformable medical image registration: Setting the state of the art with discrete methods\*." *Annual review of biomedical engineering* 13 (2011): 219-244.



# Discussion

## Clinical problems

1. Diffeomorphic image registration for pre-term preadolescent rabbit brain imaging, cross sectional studies.
2. Mosaicing of fetoscopic videos: distortion correction.
3. Landmark tracking in fetoscopic analysis of the placenta.

## Methodological problems

4. Efficient computation of the log-composition and Lie logarithm map with improved parallel transport.
5. Computation of the Lie exponential: exponential integrators applied to stochastic deformation models.
6. Computation of the Lie exponential: EDT RK4, in medical imaging.
7. Discrete optimisation methods (MRF) to optimise SVF-based energy function.

...Deadline: upgrade in **6 months!**